Exercise 17

For the following exercises, determine whether there is a minimum or maximum value to each quadratic function. Find the value and the axis of symmetry.

$$f(x) = 4x^2 + x - 1$$

Solution

Begin by factoring the coefficient of x^2.

$$f(x) = 4\left(x^2 + \frac{1}{4}x - \frac{1}{4}\right)$$

In order to write this quadratic function in vertex form, it's necessary to complete the square, which makes use of the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = \frac{1}{4}$, which means $B = \frac{1}{8}$ and $B^2 = \frac{1}{64}$. Add and subtract $\frac{1}{64}$ on the right side within the parentheses and use the identity so that x appears in only one place.

$$\begin{aligned} f(x) &= 4\left[\left(x^2 + \frac{1}{4}x + \frac{1}{64}\right) - \frac{1}{4} - \frac{1}{64}\right] \\ &= 4\left[\left(x + \frac{1}{8}\right)^2 - \frac{17}{64}\right] \\ &= 4\left(x + \frac{1}{8}\right)^2 - \frac{17}{16} \end{aligned}$$

Therefore, the vertex of the parabola is $\left(-\frac{1}{8}, -\frac{17}{16}\right)$. The axis of symmetry is $x = -\frac{1}{8}$, and the minimum (because the coefficient of x^2 is positive) is $y = -\frac{17}{16}$.